Adaptive Arrays and Tracking

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Abstract Adaptive arrays and tracking share many concepts, mathematical tools, practical issues, and algorithms. For example, ill-conditioning of the sample covariance matrix for adaptive arrays and ill-conditioning of the covariance matrix in a Kalman filter are both serious problems that can be mitigated by the same set of about a dozen methods, including Tychonoff regularization (called "diagonal loading" in adaptive arrays), factorization of the covariance matrix, using principal coordinates or approximately principal coordinates, etc. The basic mathematics of Kalman filters and adaptive arrays includes linear algebra and probability theory, but more specifically, Kalman filters and adaptive arrays use essentially the same matrix inversion formula.

Multiple hypothesis tracking is the method of choice nowadays in tracking, and it could be applied to adaptive arrays and STAP for different types of jamming, clutter, and targets. In particular, there is no reason these days to settle for only one adaptive antenna pattern, but rather we could have a bank of ten or more such adaptive antenna patterns for the same batch of data combined adaptively using standard Bayesian methods. Sample covariance matrix estimation in adaptive arrays could benefit from robust multiple hypothesis algorithms developed for tracking. Adaptive array design and STAP can be viewed as nonlinear estimation problems, which suggest that adaptive arrays could profit by using the powerful and elegant exponential family of probability densities, which includes the multivariate Gaussian density as a special case. In particular, exact nonlinear filters, which generalize the Kalman filter, have been developed using the exponential family over the last two decades. Markov random fields, which are relevant to adaptive arrays, also correspond to the exponential family, according to the Hammersley-Clifford theorem.

Tracking and adaptive arrays share a long list of common problems and solutions: Cramer-Rao bounds, fast real-time algorithms, optimal waveform (and/or array) design, multipath, unresolved targets, uncertainty in covariance matrices, non-Gaussian errors, nonlinearities, etc. In all of these areas, there is potential for cross-fertilization beyond adaptive array experts casually reading the tracking literature, and beyond tracking experts occasionally attending ASAP. The adaptive array literature is focused almost exclusively on sum patterns, but angle measurements, using adaptive monopulse antenna patterns, are crucial for many tracking applications. New algorithms developed by tracking experts for mitigating the effects of unresolved targets and multipath on monopulse can be applied profitably to adaptive arrays. A novel algorithm developed by tracking experts that significantly enhances the probability of detection of targets using monopulse data (combined with the usual sum pattern data) can also be applied to adaptive arrays.

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Adaptive Arrays and Tracking

Fred Daum

17 March 2004

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Math tools	Issues
Linear algebra Probability & statistics Optimization theory	Computational Complexity Accuracy & Robustness Ill-conditioning Non-Gaussian Errors & nonlinearity

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Adaptive Arrays & Tracking are not the same!

Tracking	Adaptive Arrays
Time: x(t)	Space & time: x(t,…)
Long time scale	Short time scale
EKF: Theoretical covariance matrix	SMI: Sample covariance matrix
Target & noise errors	RF & IF & A/D errors & propagation, etc.

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Math tools	Issues	Algorithms
Linear algebra Probability & statistics Optimization theory	Computational Complexity Ill-conditioning Accuracy & robustness Non-Gaussian Errors & nonlinearity	Least squares: Kalman filter & SMI Multiple hypotheses & multiple models Nonlinear filters

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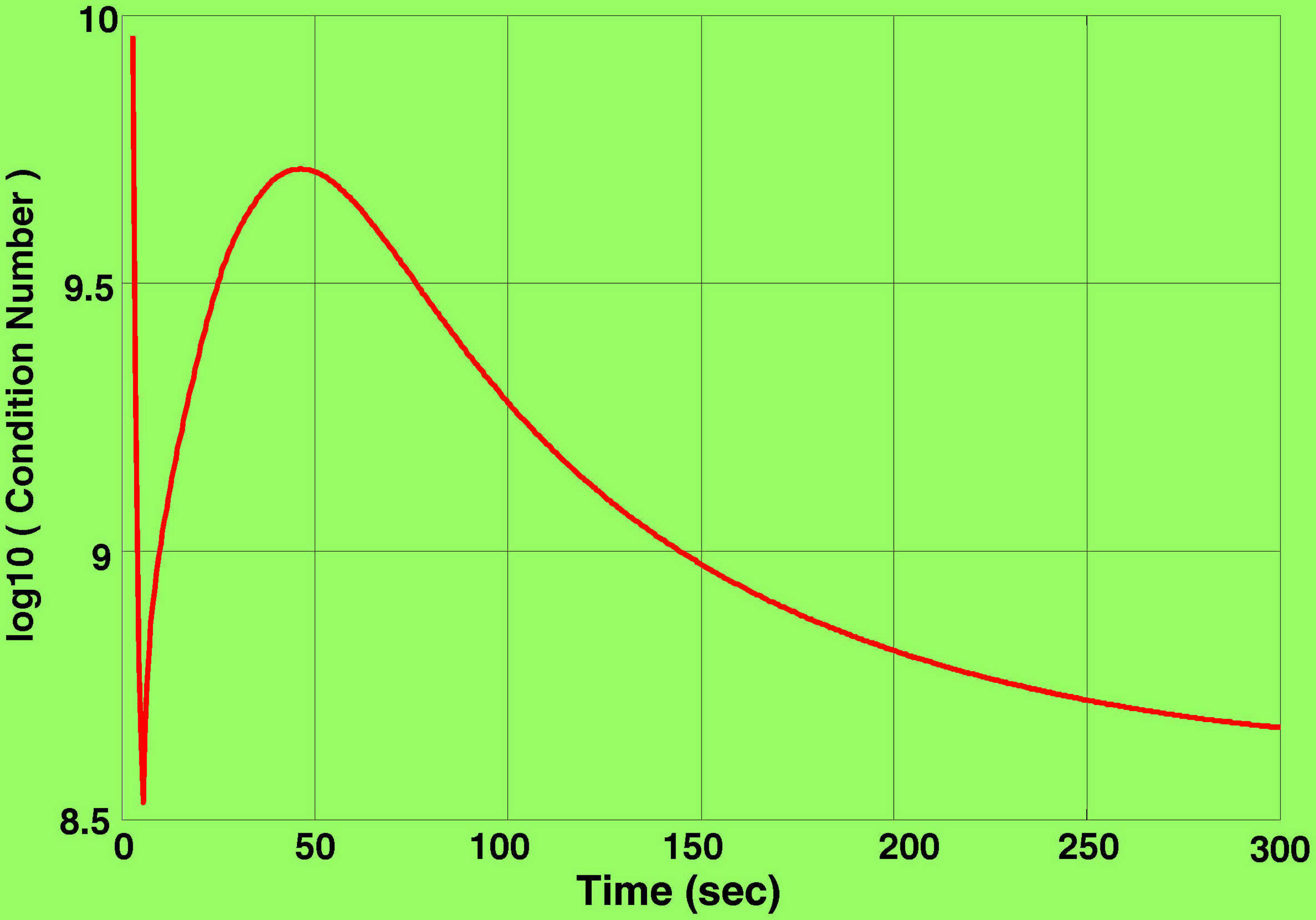
Log of Condition Number of Kalman Filter vs. Time



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Mitigation of Ill-Conditioning

- Double or higher precision arithmetic
- Tikhonov regularization (aka "ridge analysis") aka "diagonal loading"
- Symmetrization of covariance matrix
- Covariance factorization (e.g. square root)
- Principal coordinates or nearly principal coordinates.
- Decoupled or quasi-decoupled covariance
- Increase process noise covariance matrix
- Seven other dubious methods (see Daum, IEEE Trans. Automatic Control, March 1983)

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Data Association

Tracking	Adaptive Arrays
A good way to ruin the performance of a Kalman filter is to put the wrong data into it.	A good way to ruin the performance of SMI is to put the wrong data into it.
Which radar measurements came from which physical targets?	Which radar data came from the targets & which came from the jammers or clutter?

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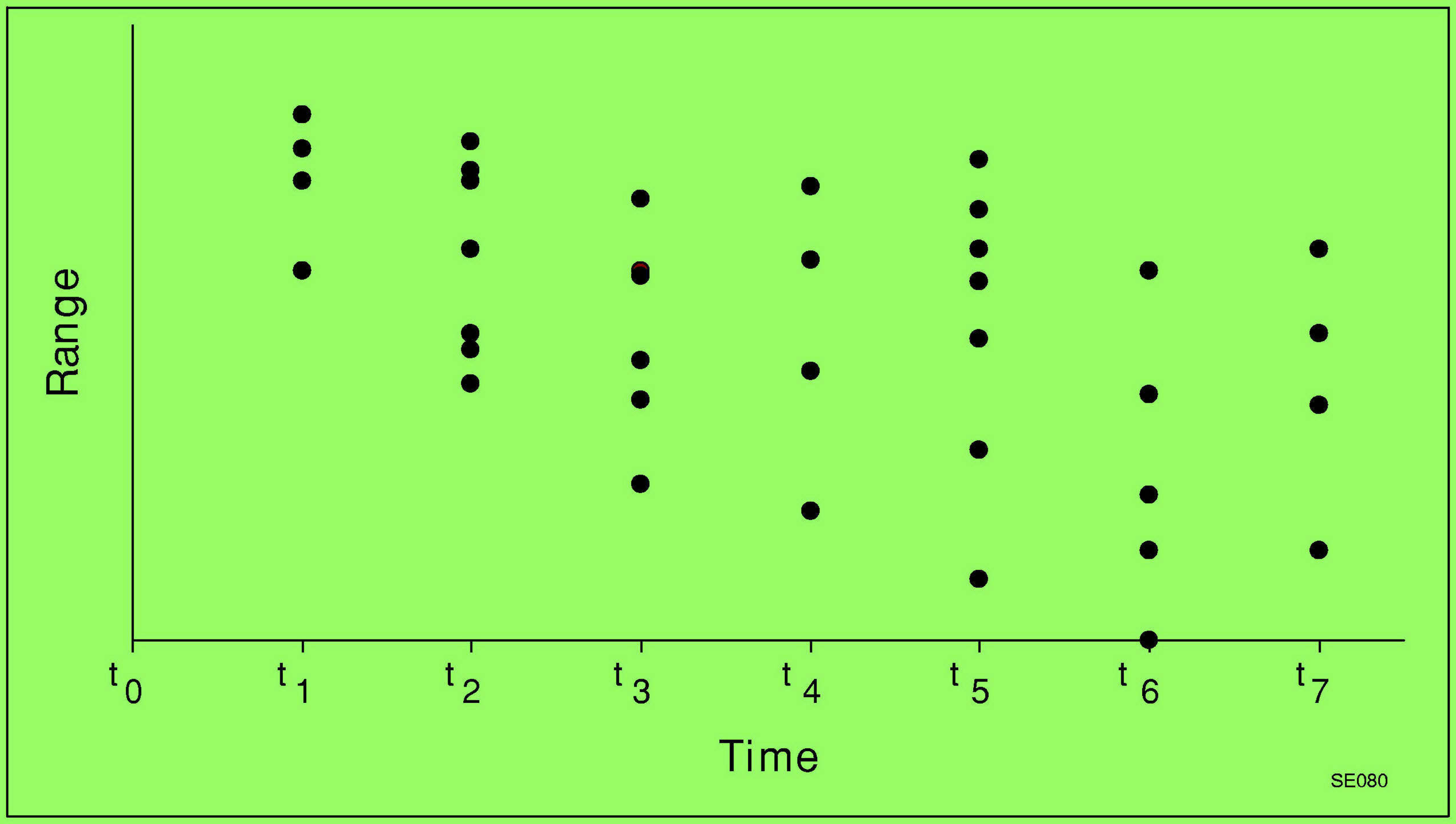
Example of Data Association Problem



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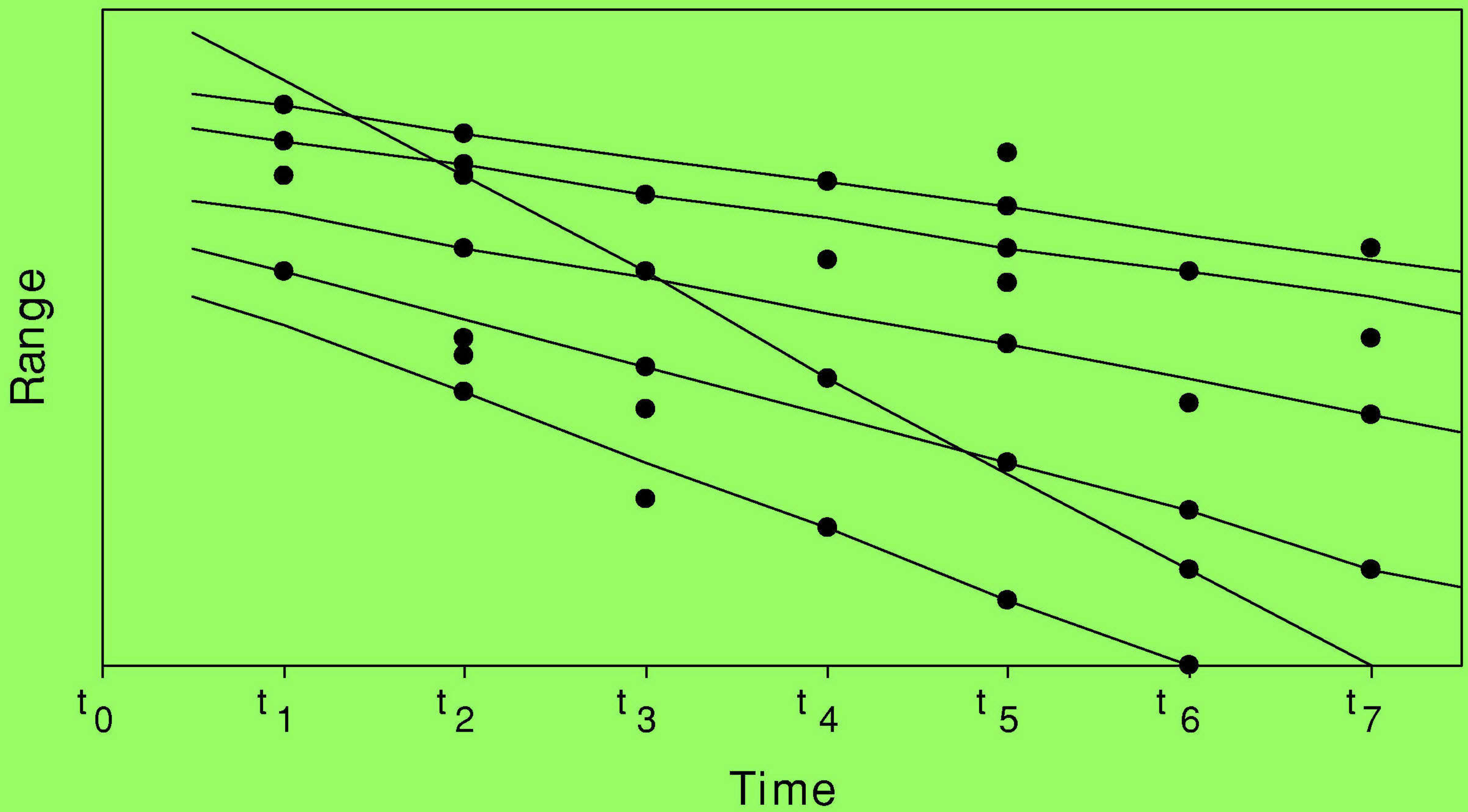
Solution to Data Association Problem



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Comparison of Multiple Target Tracking Algorithms (part 1 of 3)

Formalism or Algorithm	Time Horizon Considered (No. of Samples)	Number of Data Association Hypotheses	Unresolved Data Modeled in Algorithm	Relative Performance in Dense Multiple Target Environments		Computational Complexity	
				Unresolved Data	Resolved Data	Exact Solution	Approximate Solution
Nearest Neighbor	1	1	No	Poor	Poor	Low	Low
Nearest Neighbor - M	1	1	Yes	Fair	Poor	Low	Low
Probabilistic Data Association (PDA)	1	1	No	Poor	Fair	Low	Low
Joint Probabilistic Data Association (JPDA)	1	1	No	Fair	Good	Exp	Medium
JPDAM	1	1	Yes	Good	Good	Exp	Medium
Nearest Neighbor JPDA	1	1	No	Fair	Good to Excellent	Poly	Low
Assignment	1	1	No	Fair	Good to Excellent	Poly	Medium

Comparison of Multiple Target Tracking Algorithms (part 2 of 3)

Formalism or Algorithm	Time Horizon Considered (No. of Samples)	Number of Data Association Hypotheses	Unresolved Data Modeled in Algorithm	Relative Performance in Dense Multiple Target Environments		Computational Complexity	
				Unresolved Data	Resolved Data	Exact Solution	Approximate Solution
Dynamic Programming (Viterbi)	Many	1	No	Poor	Good	Poly	Medium
Hough Transform	Many	1	No	Fair	Good	Poly	Medium
Multiple Hypothesis Tracking (MHT)	Many	Many	No	Good	Optimal	Exp	High
MHT-M	Many	Many	Yes	Best	Excellent	Exp	High
Morefield	Many	Many	No	Fair	Excellent	Exp	High
Symmetric Measurements EKF	Many	Many	No	?	Good	Poly	High
Symmetric Measurements Nonrecursive	Many	Many	No	?	Excellent	Exp	High

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Comparison of Multiple Target Tracking Algorithms (part 3 of 3)

Formalism or Algorithm	Time Horizon Considered (No. of Samples)	Number of Data Association Hypotheses	Unresolved Data Modeled in Algorithm	Relative Performance in Dense Multiple Target Environments		Computational Complexity	
				Unresolved Data	Resolved Data	Exact Solution	Approximate Solution
Branching	Many	Many	No	Fair	Excellent	Bounded	Med. to High
Branching - M	Many	Many	Yes	Good	Excellent	Bounded	Med. to High
Multi-Dimensional Assignment	Many	Many	No	Good	Excellent	-	High
Multi-Dimensional Assignment - M	Many	Many	Yes	Excellent	Excellent	-	High
Exact N-Best Hypotheses	Many	N	No	Good	Excellent	-	Medium

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Multiple Hypotheses & Models

Tracking	Adaptive Arrays
Multiple Kalman filters combined adaptively (e.g., IMM) MHT for data association (see Sam Blackman's book)	Multiple antenna patterns combined adaptively. Different types of jamming, RFI, clutter, multipath; number of closely spaced targets; array hardware health; ambiguities in range, Doppler, angle, phase, etc.

Varieties of Nonlinear Filters

- Extended Kalman filters
- Unscented EKFs
- Particle filters
- Quasi-Monte Carlo hybrid particle filters
- Daum filters
- Non-recursive filters (zero process noise)
- Non-recursive filters (non-zero process noise)
- Semi-recursive filters
- Bayes' rule & numerical solution of the Fokker-Planck equation
- Other

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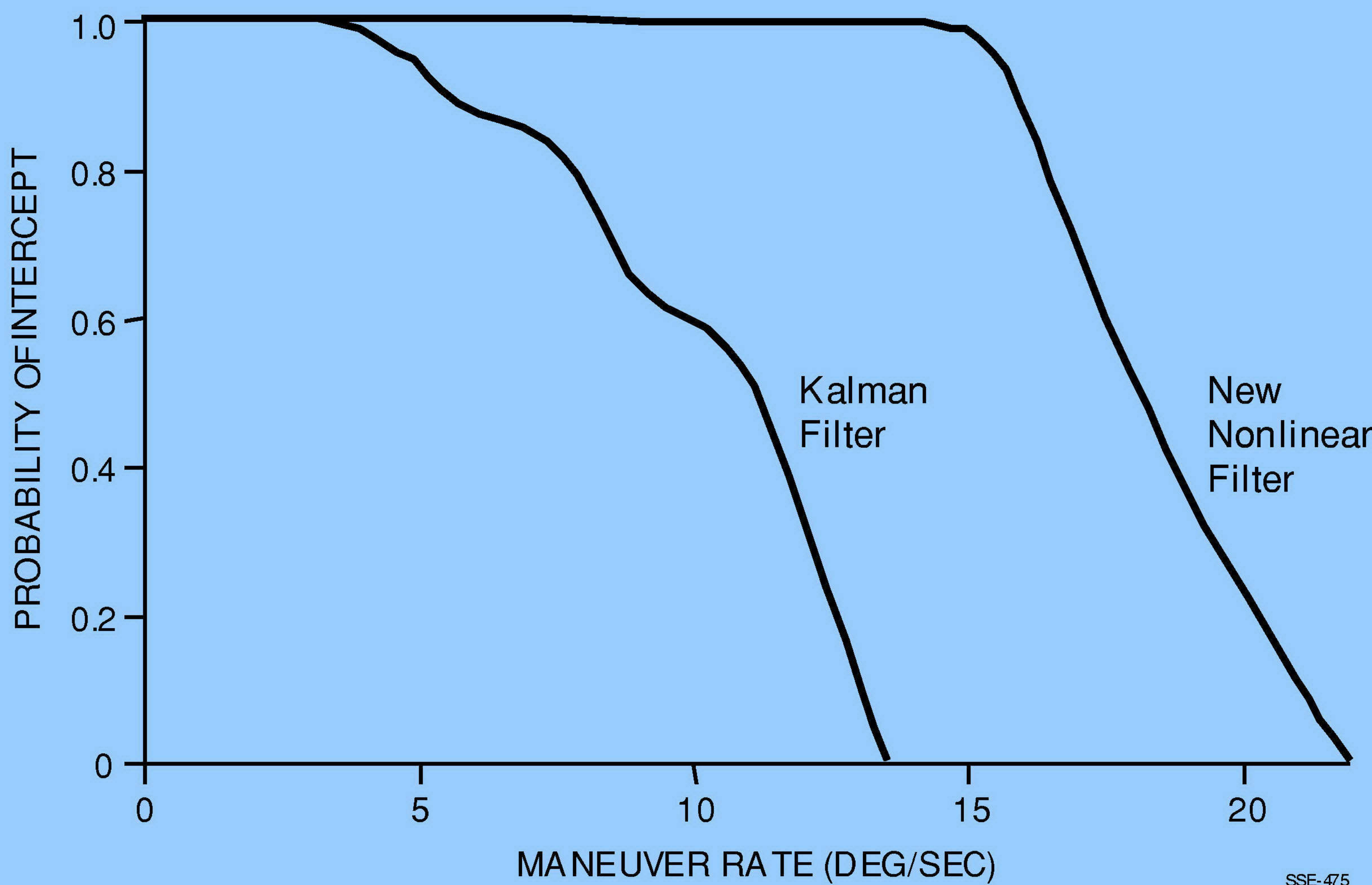
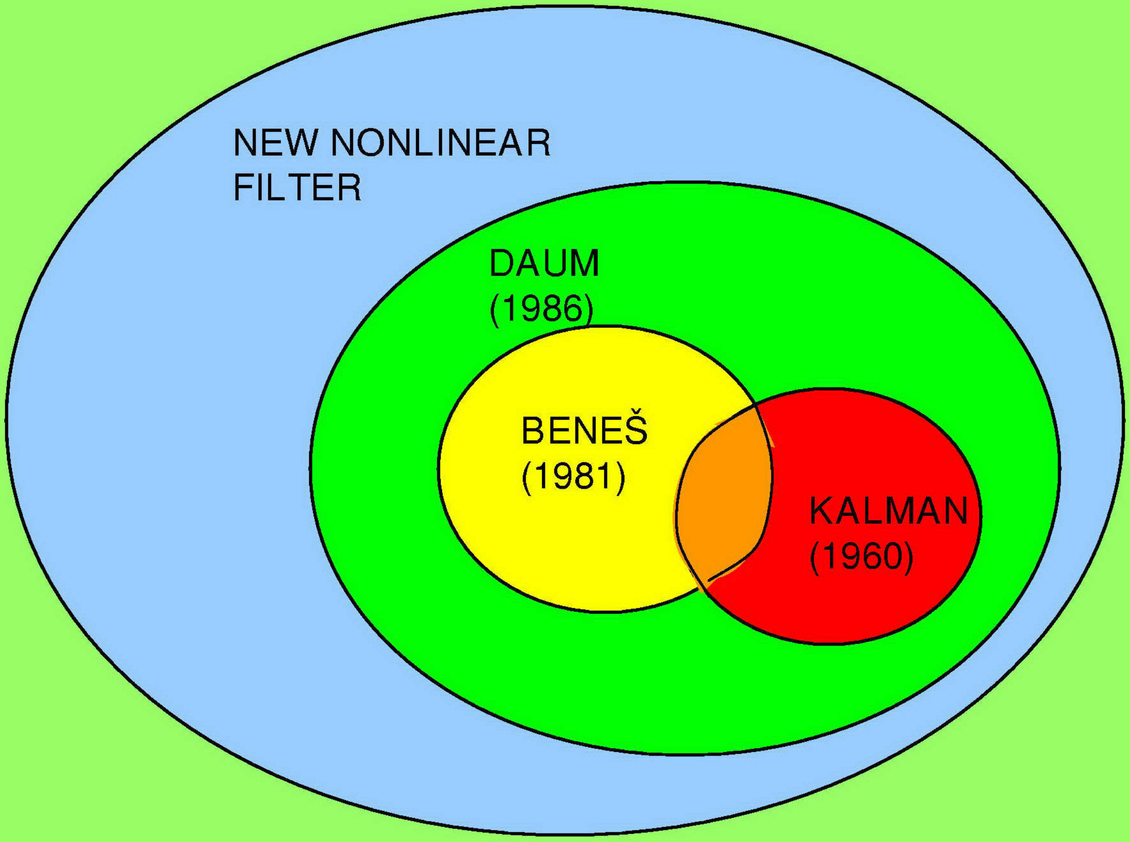
Nonlinear filter vs. Kalman filter



G. C. Schmidt AIAA J Guidance, Control & Dynamics March-April 1993

Relationship of Filters



Exact Recursive Filters

Filter	Conditional Density	Comments
1. Kalman (1960)	η = Gaussian	$\eta(x,t) = \exp\left[-\frac{1}{2}(x-\bar{x})^T C^{-1}(x-\bar{x})\right]$
2. Beneš (1981)	$\eta \exp\left[\int^x f(x)dx\right]$	$dx = f(x)dt + dw \quad$ for $f(x) = \text{gradient} = \frac{\partial\phi}{\partial x}$ $P_{ss}(x) = \exp\left[2\int^x f(x)dx\right]$
3. Daum (1986)	$\eta P_{ss}^{\alpha}(x)$	$P_{ss}(x)$ = steady state solution of Fokker-Planck equation α = constant
4. Daum (1986)	$\eta q^{\alpha}(x,t)$	$q(x, t)$ = transient solution of Fokker-Planck equation α = constant
5. Daum (1986)	$\eta Q(x\ t)$	$Q(x, t)$ = solution of a certain PDE
6. New Nonlinear Filter	$p(x,t)\exp\left[\theta^T(x,t)\psi(Z_k,t)\right]$	Exponential Family of probability densities

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Comparison of New Nonlinear Filter and Kalman Filter

Item	Kalman Filter	New Nonlinear Filter
1. Conditional density	Gaussian	Exponential Family
2. Sufficient statistic	$\hat{x}$ = conditional mean P = covariance matrix	ψ = vector of dimension M
3. Prediction equations	$\frac{d\hat{x}}{dt} = F\hat{x}$ $\frac{dP}{dt} = FP + PF^T + Q$	$\frac{d\psi}{dt} = A\psi + \psi^T B\psi$
4. Measurement update equations	$\hat{x}_k = \bar{x}_k + P_k H_k^T R_k^{-1}(z_k - H_k \bar{x}_k)$ $P_k = M_k - M_k H_k^T (H_k M_k H_k^T + R_k)^{-1} H_k M_k$	$\psi_k = \overline{\psi}_k + c(z_k, t_k)$
5. Dimension of sufficient statistic	n + n(n + 1)/2	M
6. Dynamic model	linear	nonlinear
7. Measurement model	linear	nonlinear
8. Measurement noise	Gaussian	non-Gaussian
9. Measurements	discrete time	discrete time
10. Dynamics	continuous time	continuous time
11. Computational complexity	numerical integration of ODE	numerical integration of ODE
12. Approximation method	linearization	finite sum decomposition

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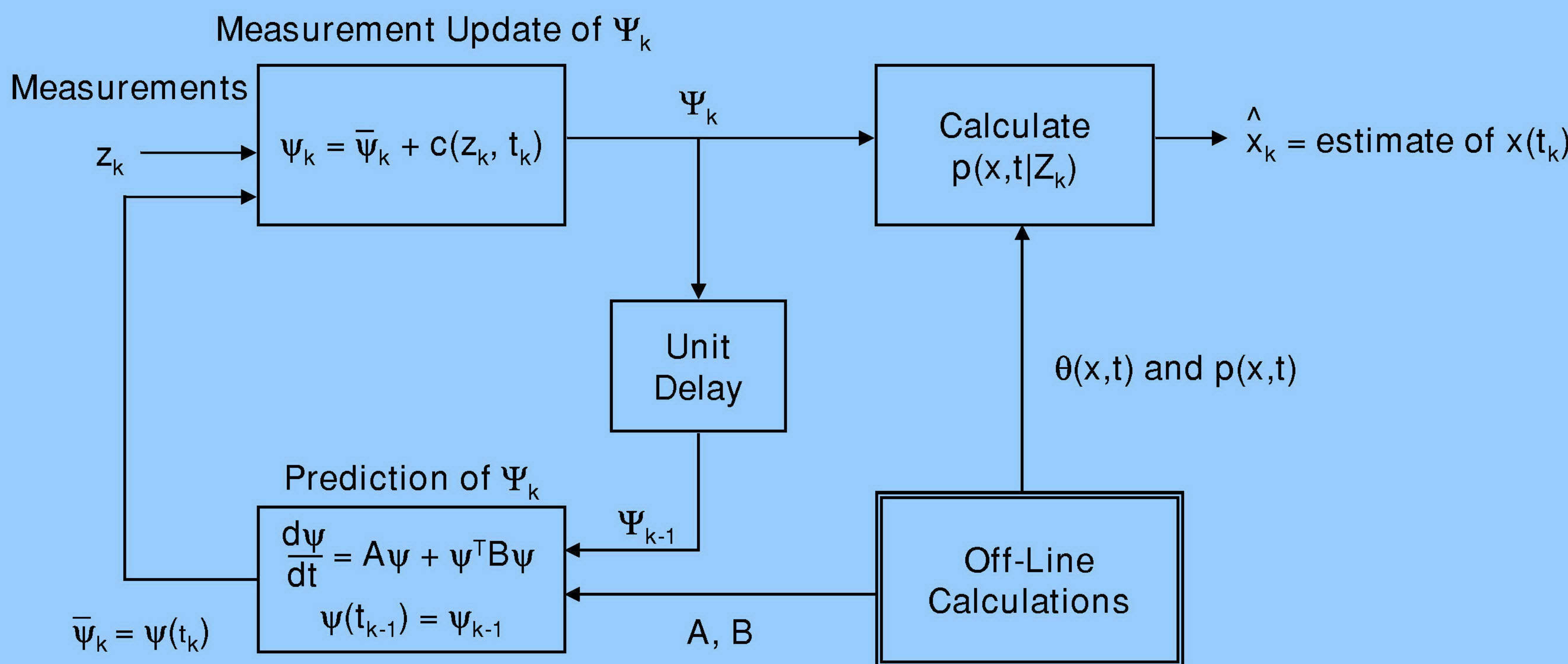
New Nonlinear Filter



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Generalized Darmois-Koopman-Pitman Theorem

For smooth nowhere vanishing densities, a fixed finite dimensional filter exists if and only if the conditional density is from an exponential family:

$$p(x, t|Z_k) = a(x, t)b(Z_k, t) \exp [\theta^T (x, t) \psi (Z_k, t)]$$

W.O.L.O.G.

$$\boxed{p(x, t|Z_k) = p(x, t) \exp [\theta^T(x, t) \psi (Z_k, t)]}$$

x = state vector (n-dimensional)

t = time

Z_k = set of all measurements up to and including time t_k

$\psi(Z_k, t)$ = sufficient statistic

$p(x, t)$ = unconditional density of x at time t

Assertion About Particle Filters

"Particle filtering methods beat the curse of dimensionality as the rate of convergence is independent of the state dimension."

Dan Crisan & Arnaud Doucet

(IEEE Trans. Signal Processing March 2002)

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Folk Theorem About Monte Carlo Integration

$$\sigma_X = \sigma / \sqrt{N}$$

Assumptions:

(1) Integrand is known exactly

(2) N statistically independent samples

(3) Efficient sampling from the relevant probability density

(4) Certain regularity conditions

Definition of Dimension Free Error

$$r = E\left[(x - \hat{x})^* J(x - \hat{x})\right]/d$$

in which

x = state vector to be estimated

$\hat{x}$ = estimate of x

d = dimension of x

J = inverse of the estimation error covariance matrix for the optimal filter

$E(.)$ = expected value of (.)

$(.)^*$ = transpose of (.)

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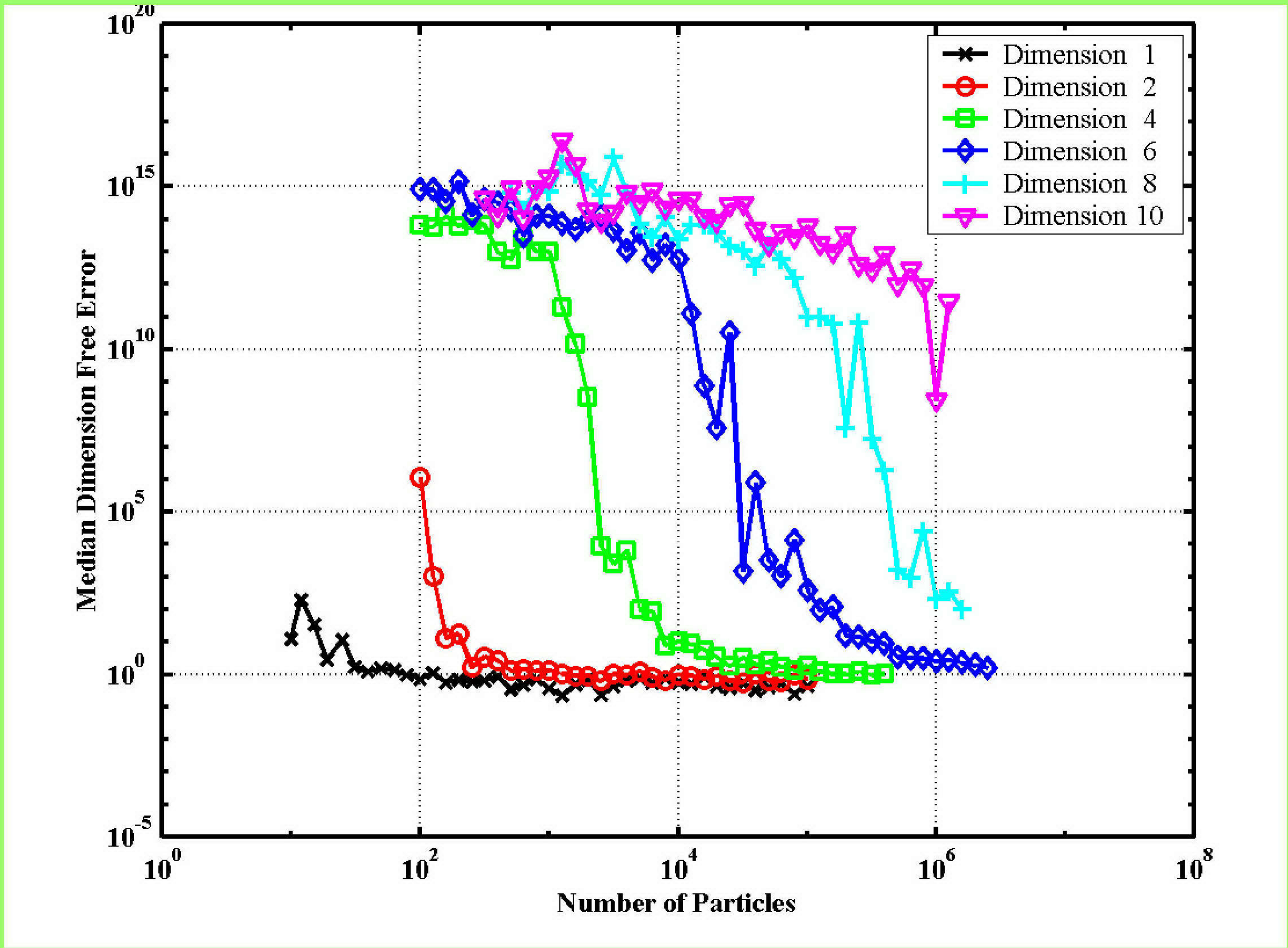
Dimension Free Error vs. Number of Particles for PV-PF



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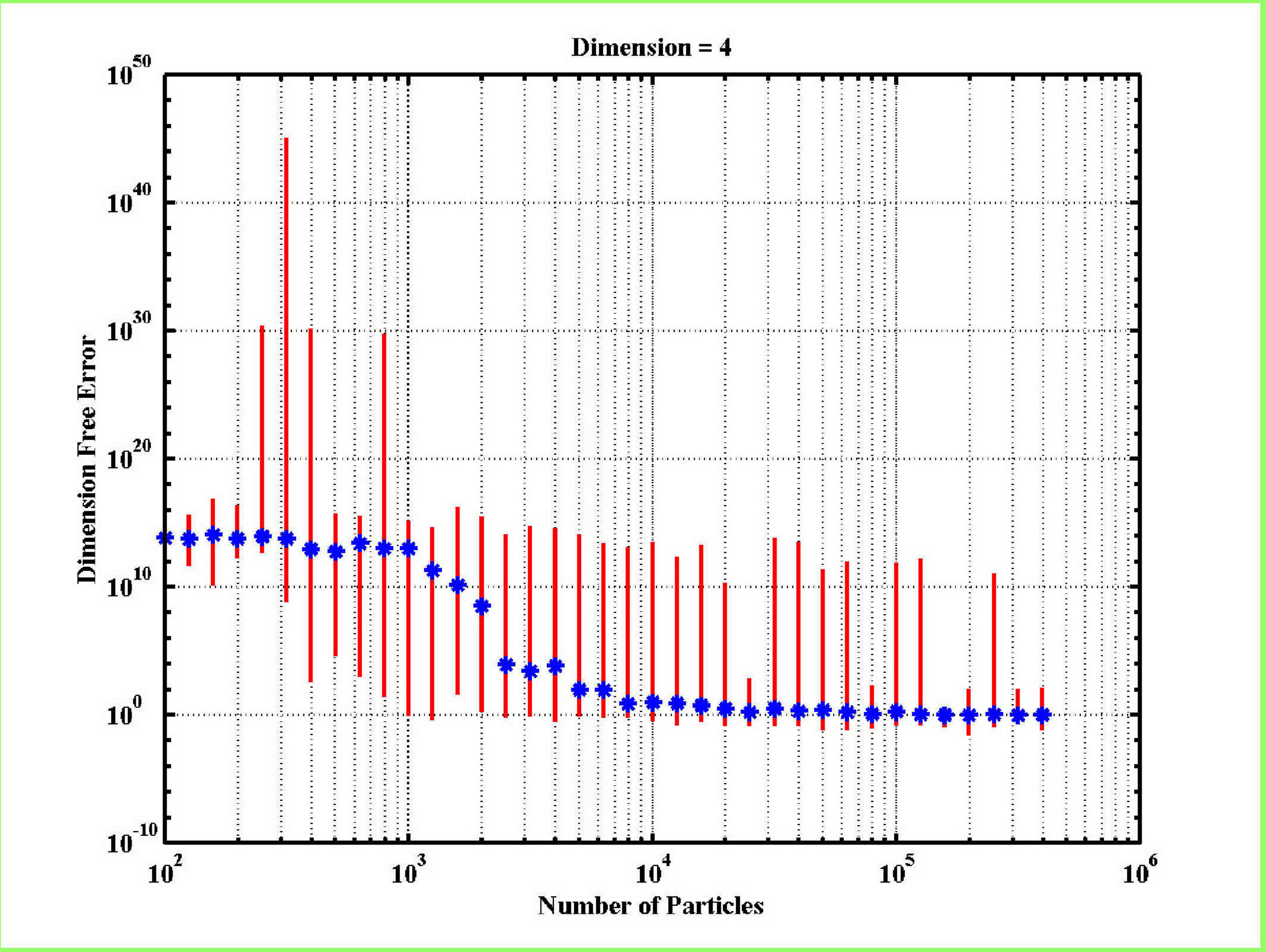

Dimension = 4
Dimension Free Error
Number of Particles
10^{50}
10^{40}
10^{30}
10^{20}
10^{10}
10^{0}
10^{-10}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}

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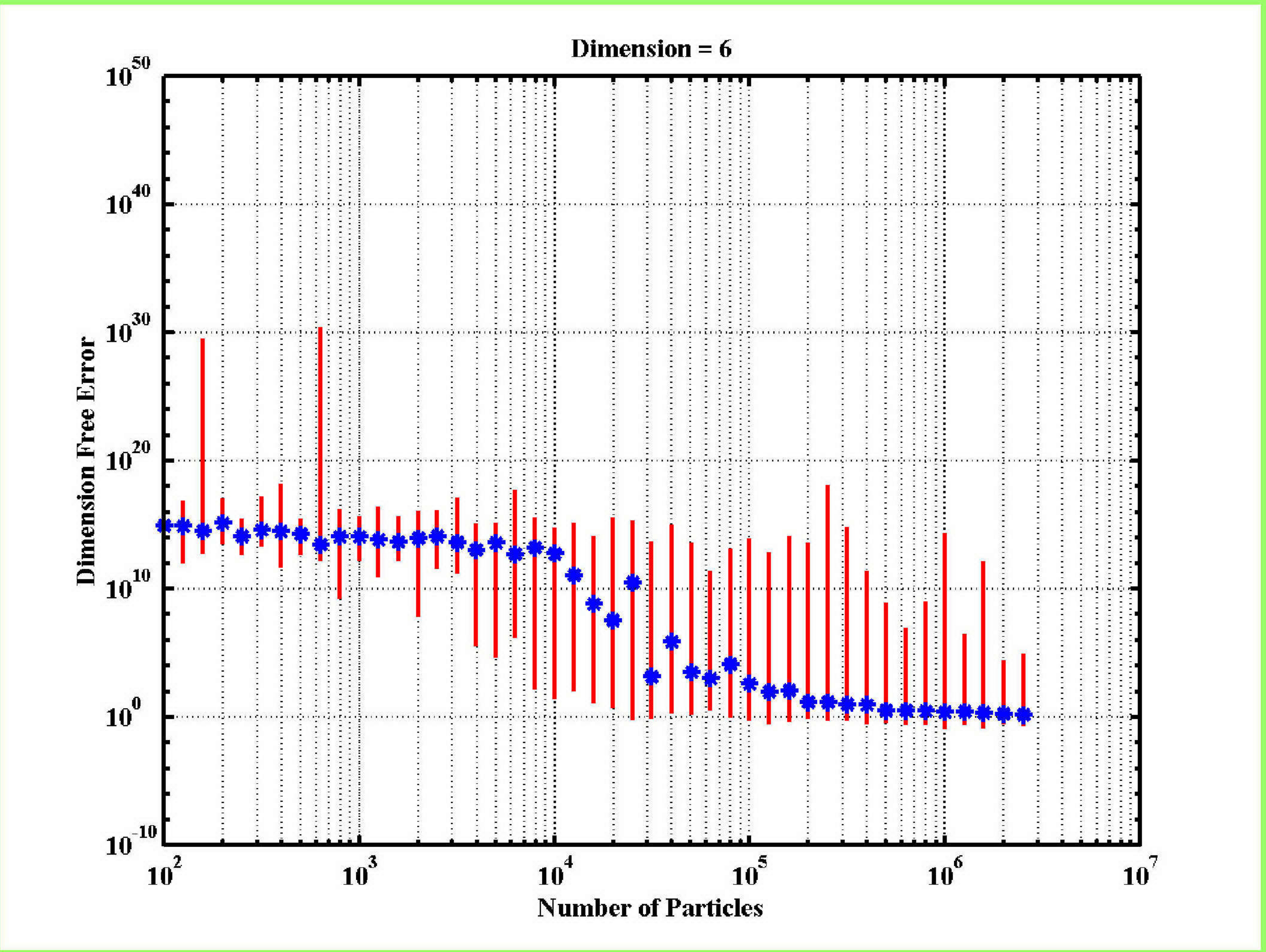

Dimension = 6
10^{50}
10^{40}
10^{30}
10^{20}
10^{10}
10^{0}
10^{-10}
Dimension Free Error
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
Number of Particles

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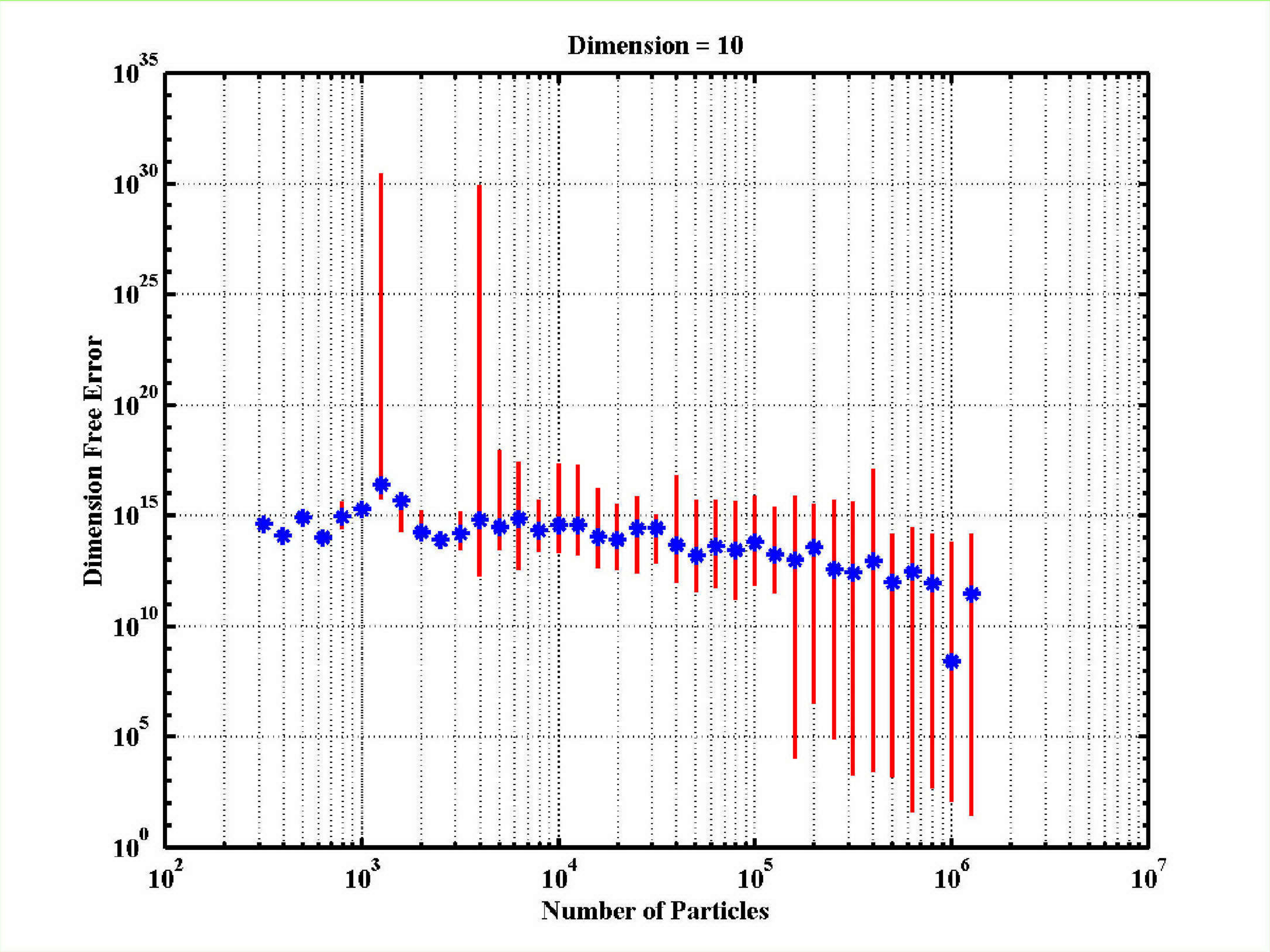

Dimension = 10
Dimension Free Error
Number of Particles
10^{35}
10^{30}
10^{25}
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}

Quote from Doucet’s Website

QUESTION: When you say “particle filters beat the curse of dimensionality,” what do you mean?

ANSWER: A rather unfortunate expression we’ve used ourselves to look good.

Multiple Choice Quiz

What is the approximate ratio (r) of the volume of the unit sphere to the volume of the smallest cube that contains it for d = 10 dimensions?

(a) r = 0.5

(b) r = 0.1

(c) r = 0.01

(d) r = 0.002

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Another Multiple Choice Quiz

What is the approximate ratio (r) of the volume of the unit sphere to the volume of the smallest cube that contains it for d = 100 dimensions?

(a) $r = 10^{-4}$

(b) $r = 10^{-6}$

(c) $r = 10^{-10}$

(d) $r = 10^{-69}$

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Simple Back-of-the-Envelope Formula

$$T = cd^{\alpha+1} \big/ \sigma_x^2$$

Assumptions:

(1) The filtering problem has a "vaguely Gaussian" conditional probability density (pdf)

(2) The PF is designed & tuned to avoid particle collapse

(3) The PF is designed & tuned to sample the pdf efficiently

(4) The N Monte Carlo samples are statistically independent

(5) Certain regularity conditions

(6) Computational complexity (per particle) is on the order of d^{α}.

Varieties of Particle Filters

Item	Varieties
1. Proposal density	Extended Kalman filter (Gaussian) Unscented filter (Gaussian) Daum filter (exponential family) Gaussian sums Sums of exponential family Other
2. Sampling	Rejection Naïve importance sampling Metropolis-Hastings importance sampling Gibbs sampling Other
3. Re-sampling	Every update Sparse Adaptive Other
4. Representation of conditional density	Particles Smooth Kernels (F. Le Gland, et al) Other
5. Generation of samples	Deterministic grid Monte Carlo Quasi-Monte Carlo Hybrid Other
6. Variance reduction methods	Stratified Sampling Rao-Blackwellization Control variates Antithetic variables Other

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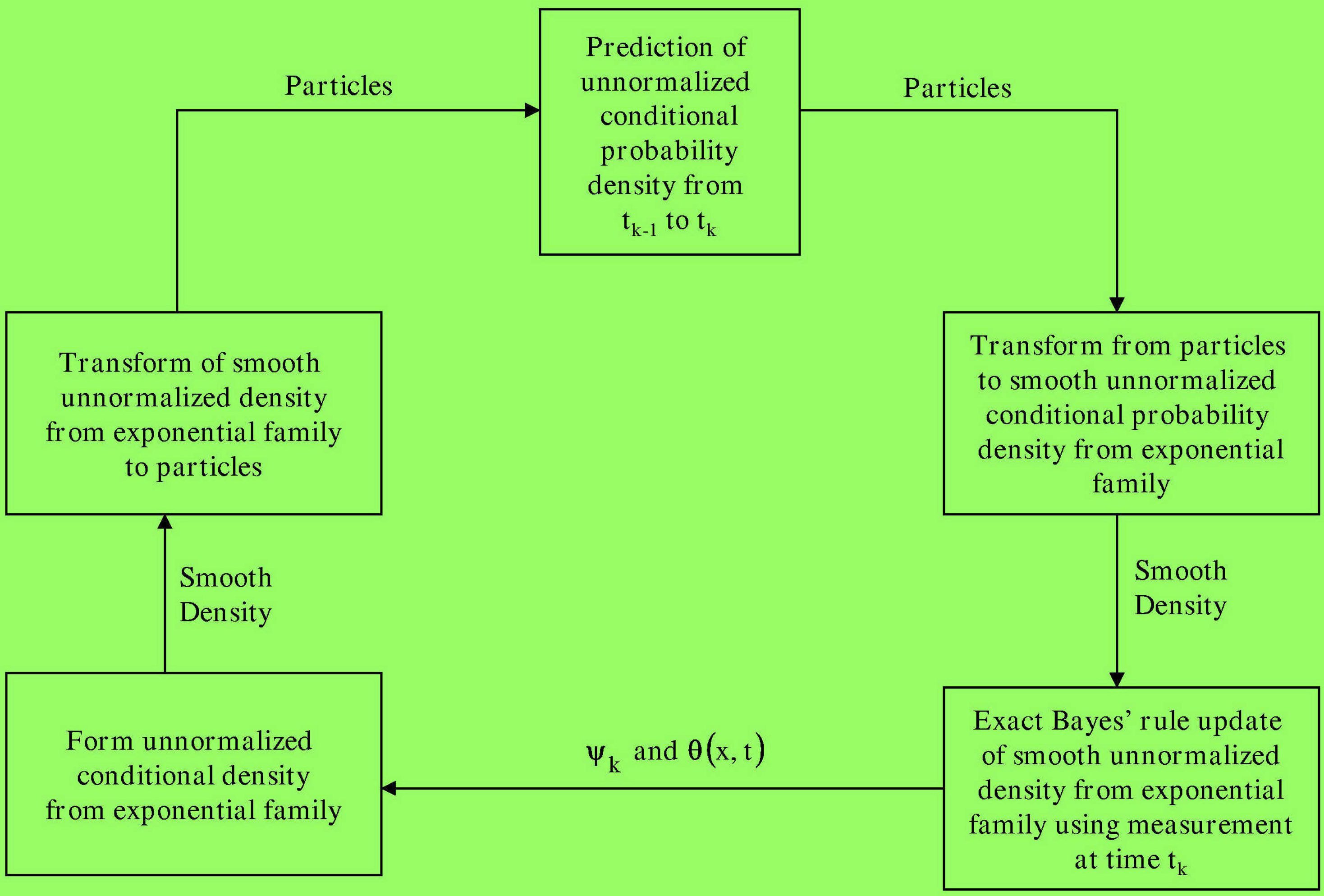
Quasi-Monte Carlo Hybrid Particle Filter



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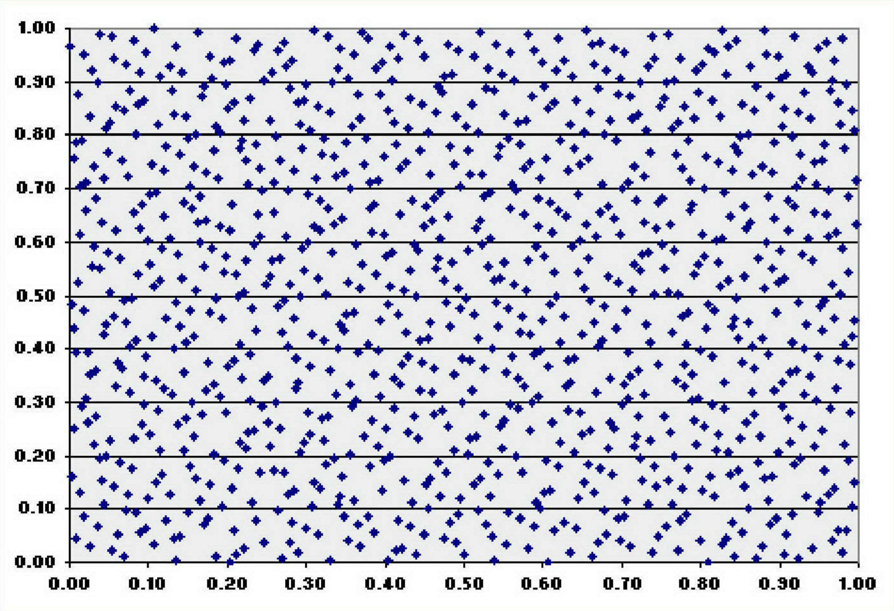
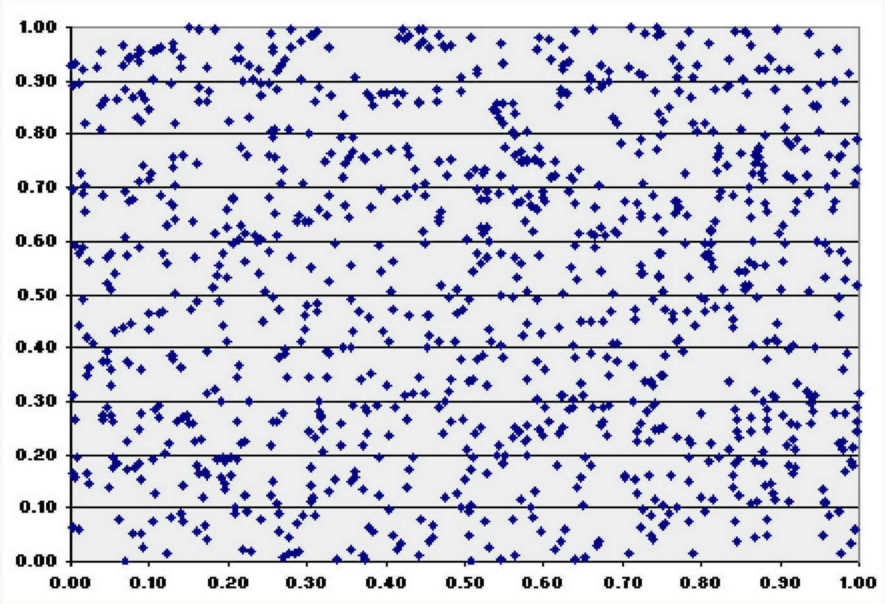
Quasi-Monte Carlo Samples



Monte Carlo Samples



(Ref: http://www.puc-rio.br/marco.ind/quasi_mc.html)

Curse of Dimensionality and Particle Filters

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	Bad Estimation Problem	Nice Estimation Problem
Plain vanilla particle filters	Curse of dimensionality	Curse of dimensionality
Carefully designed particle filter with bells & whistles	Curse of dimensionality	Avoids curse of dimensionality

No particles were harmed in the production of this talk.